Scalable parametric estimation procedure for kinematic models of N-trailer vehicles

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Outline

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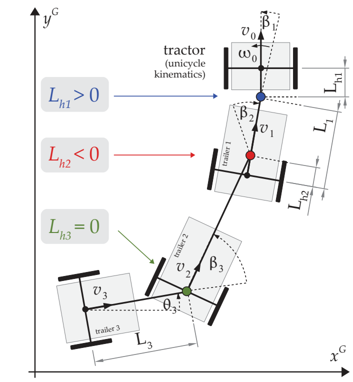
Why to consider N-trailer vehicles?

- Popular means of freight transportation
- Increasing demand in long multi-articulated vehicles (High Capacity Vehicle(s) = HCV)
- Maneuvering with N-trailers – difficult and burdening for a human-driver
- Automated or semi-automated (intelligent) N-trailers – the future of HCV

Photos taken from: M. M. Michałek: Scalable parametric-identification procedure for kinematics of automated N-trailer vehicles, IEEE TVT, 2024

Kinematics of N-trailers – two types of trailer hitching



L_i – length of the ith trailer
L_{hi} – hitching offsets of the ith joint

- ON-AXLE hitching: $L_{hi} = 0$
- OFF-AXLE hitching: $L_{hi} \neq 0$

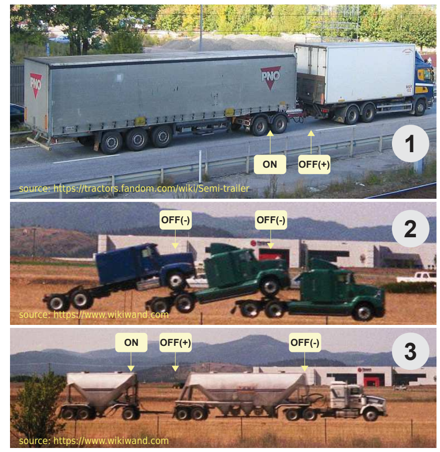
Proposed classification [M:12]:

$N_h = 0$	$0 < N_h < N$	$N_h = N$
SNT	*GNT*	*nSNT*

N_h – the number of off-axle hitching

SNT = Standard N-Trailer
GNT = Generalized N-Trailer
nSNT = non-Standard N-Trailer

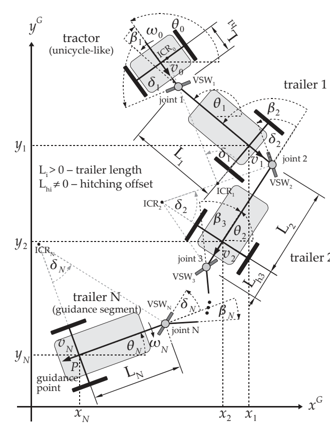
[M:12] M.M. Michałek: Application of the VFO method to set-point control for the N-trailer vehicle with off-axle hitching, Int. Journal of Control, 2012



Hitching types substantially influence maneuverability of N-trailers!

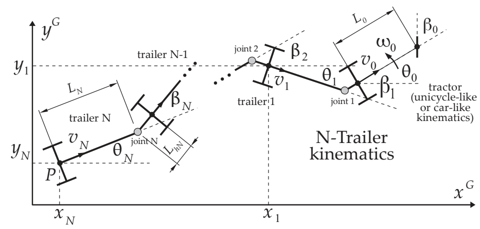
Parametric identification of kinematic models for N-trailers – motivation [M:22,M:24]

- Vehicle's behaviour dominated by kinematics in low-speed maneuvering
- Kinematic model can be useful for purposes of:
  - vehicle localization
  - automatic planning of maneuvers
  - low-speed automatic feedback control (automatic guidance)
- Kinematic parameters of automated N-trailers can be:
  - unknown (picking-up an unknown trailer or a chain of trailers)
  - uncertain (due to a slippage of wheels, due to unmodelled effects like backlash in joints etc.)
  - time-varying (when cornering with multi-axle trailers of fixed wheels)
- Parametric identification of N-trailer kinematics seems practically useful



[M:22] M. M. Michałek: Scalable procedure of parametric estimation for N-trailer kinematics, IEEE SMC, 2022
[M:24] M. M. Michałek: Scalable parametric-identification procedure for kinematics of automated N-trailer vehicles, IEEE TVT, 2024

Joint-angle kinematics of nonholonomic N-trailer vehicles with fixed wheels



- Kinematic parameters for $i = 1, \ldots, N$:
  - $L_i > 0$ (trailer's length)
  - $L_{hi} \in \mathbb{R}$ (hitching offset)
- N-trailer classification (N_h = number of $L_{hi} \neq 0$):
  - SNT ($N_h = 0$)
  - GNT ($0 < N_h < N$)
  - nSNT ($N_h = N$)
- Kinematic control input: $\boldsymbol{u}_0 = [\omega_0 \ v_0]^\top \subset \mathbb{R}^2$

Generic joint-angle kinematics for N-trailers with fixed wheels:

$$\underbrace{\begin{bmatrix} \beta_1^{(1)} \\ \beta_2^{(1)} \\ \beta_3^{(1)} \\ \vdots \\ \beta_N^{(1)} \end{bmatrix}}_{\boldsymbol{\beta}^{(1)}} = \underbrace{\begin{bmatrix} \boldsymbol{c}^\top \boldsymbol{\Gamma}_1(\beta_1,\boldsymbol{\eta}_1) \\ \boldsymbol{c}^\top \boldsymbol{\Gamma}_2(\beta_2,\boldsymbol{\eta}_2)\,\boldsymbol{J}_1(\beta_1,\boldsymbol{\eta}_1) \\ \boldsymbol{c}^\top \boldsymbol{\Gamma}_3(\beta_3,\boldsymbol{\eta}_3)\,\boldsymbol{J}_2(\beta_2,\boldsymbol{\eta}_2)\,\boldsymbol{J}_1(\beta_1,\boldsymbol{\eta}_1) \\ \vdots \\ \boldsymbol{c}^\top \boldsymbol{\Gamma}_N(\beta_N,\boldsymbol{\eta}_N)\,\prod_{j=N-1}^{1} \boldsymbol{J}_j(\beta_j,\boldsymbol{\eta}_j) \end{bmatrix}}_{\boldsymbol{S}_\beta(\beta_1,\ldots,\beta_N,\boldsymbol{\eta})} \boldsymbol{u}_0 \qquad (1)$$

- β_i and $\beta_i^{(1)}$ – ith joint angle and its time derivative (i.e., $\beta_i^{(1)} \triangleq \mathrm{d}\beta_i/\mathrm{d}t$)
- $\boldsymbol{\eta} = [\boldsymbol{\eta}_1^\top \ \ldots \ \boldsymbol{\eta}_N^\top]^\top \in \mathbb{R}^N \times \mathbb{R}_+^N$ – vector of kinematic parameters
- $\boldsymbol{\eta}_i = [L_{hi} \ L_i]^\top \in \mathbb{R} \times \mathbb{R}_+$ – kinematic parameters associated with ith joint
- $\boldsymbol{\Gamma}_j(\beta_j, \boldsymbol{\eta}_j) \triangleq \boldsymbol{I}_{2\times2} - \boldsymbol{J}_j(\beta_j, \boldsymbol{\eta}_j)$, $\quad \boldsymbol{c}^\top \triangleq [1 \ 0]$
- $\boldsymbol{J}_j(\beta_j, \boldsymbol{\eta}_j)$ – velocity transformation matrix satisfying

$$\underbrace{\begin{bmatrix} \omega_i \\ v_i \end{bmatrix}}_{\boldsymbol{u}_i} = \underbrace{\begin{bmatrix} -\frac{L_{hi}}{L_i}\cos\beta_i & \frac{1}{L_i}\sin\beta_i \\ L_{hi}\sin\beta_i & \cos\beta_i \end{bmatrix}}_{\boldsymbol{J}_i(\beta_i,\boldsymbol{\eta}_i)} \underbrace{\begin{bmatrix} \omega_{i-1} \\ v_{i-1} \end{bmatrix}}_{\boldsymbol{u}_{i-1}}, \quad i = 1, \ldots, N$$

Complexity grow of joint-angle kinematics for N increasing

For the joint-angle kinematics

$$\underbrace{\begin{bmatrix} \beta_1^{(1)} \\ \beta_2^{(1)} \\ \beta_3^{(1)} \\ \vdots \\ \beta_N^{(1)} \end{bmatrix}}_{\boldsymbol{\beta}^{(1)}} = \underbrace{\begin{bmatrix} \boldsymbol{c}^\top \boldsymbol{\Gamma}_1(\beta_1,\boldsymbol{\eta}_1) \\ \boldsymbol{c}^\top \boldsymbol{\Gamma}_2(\beta_2,\boldsymbol{\eta}_2)\,\boldsymbol{J}_1(\beta_1,\boldsymbol{\eta}_1) \\ \boldsymbol{c}^\top \boldsymbol{\Gamma}_3(\beta_3,\boldsymbol{\eta}_3)\,\boldsymbol{J}_2(\beta_2,\boldsymbol{\eta}_2)\,\boldsymbol{J}_1(\beta_1,\boldsymbol{\eta}_1) \\ \vdots \\ \boldsymbol{c}^\top \boldsymbol{\Gamma}_N(\beta_N,\boldsymbol{\eta}_N)\,\prod_{j=N-1}^{1} \boldsymbol{J}_j(\beta_j,\boldsymbol{\eta}_j) \end{bmatrix}}_{\boldsymbol{S}_\beta(\beta_1,\ldots,\beta_N,\boldsymbol{\eta})} \boldsymbol{u}_0, \qquad \boldsymbol{u}_0 = \begin{bmatrix} \omega_0 \\ v_0 \end{bmatrix} \qquad (2)$$

the rows of matrix $\boldsymbol{S}_\beta(\beta_1, \ldots, \beta_N, \boldsymbol{\eta})$ take the following particular forms (using a shorter notation $\mathrm{s}_i \equiv \sin\beta_i$, $\mathrm{c}_i \equiv \cos\beta_i$):

$$\boldsymbol{c}^\top \boldsymbol{\Gamma}_1 = \left[\left(1 + \frac{L_{h1}}{L_1}\mathrm{c}_1\right) \quad \left(-\frac{1}{L_1}\mathrm{s}_1\right) \right]$$

$$\boldsymbol{c}^\top \boldsymbol{\Gamma}_2 \boldsymbol{J}_1 = \left[\left(-\frac{L_{h1}}{L_1}\mathrm{c}_1 - \frac{L_{h2}L_{h1}}{L_2L_1}\mathrm{c}_2\mathrm{c}_1 - \frac{L_{h1}}{L_2}\mathrm{s}_2\mathrm{s}_1\right) \quad \left(\frac{1}{L_1}\mathrm{s}_1 + \frac{L_{h2}}{L_2L_1}\mathrm{c}_2\mathrm{s}_1 - \frac{1}{L_2}\mathrm{s}_2\mathrm{c}_1\right) \right]$$

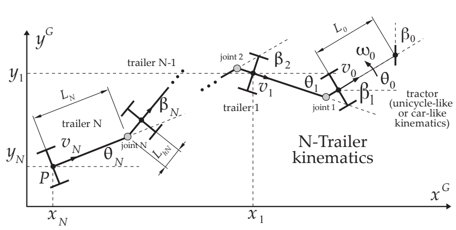
$$\boldsymbol{c}^\top \boldsymbol{\Gamma}_3 \boldsymbol{J}_2 \boldsymbol{J}_1 = \begin{bmatrix} \left(\frac{L_{h2}L_{h1}}{L_2L_1}\mathrm{c}_2\mathrm{c}_1 + \frac{L_{h1}}{L_2}\mathrm{s}_2\mathrm{s}_1 + \frac{L_{h3}L_{h2}L_{h1}}{L_3L_2L_1}\mathrm{c}_3\mathrm{c}_2\mathrm{c}_1 + \frac{L_{h3}L_{h1}}{L_3L_2}\mathrm{c}_3\mathrm{s}_2\mathrm{s}_1 + \frac{L_{h2}L_{h1}}{L_3L_1}\mathrm{s}_3\mathrm{s}_2\mathrm{c}_1 - \frac{L_{h1}}{L_3}\mathrm{s}_3\mathrm{c}_2\mathrm{s}_1\right) \\ \left(-\frac{L_{h2}}{L_2L_1}\mathrm{c}_2\mathrm{s}_1 + \frac{1}{L_2}\mathrm{s}_2\mathrm{c}_1 - \frac{L_{h3}L_{h2}}{L_3L_2L_1}\mathrm{c}_3\mathrm{c}_2\mathrm{s}_1 + \frac{L_{h3}}{L_3L_2}\mathrm{c}_3\mathrm{s}_2\mathrm{c}_1 - \frac{L_{h2}}{L_3L_1}\mathrm{s}_3\mathrm{s}_2\mathrm{s}_1 - \frac{1}{L_3}\mathrm{s}_3\mathrm{c}_2\mathrm{c}_1\right) \end{bmatrix}^\top$$

$$\vdots$$

$\boldsymbol{c}^\top \boldsymbol{\Gamma}_N \boldsymbol{J}_{N-1} \ldots \boldsymbol{J}_1 = \ldots$ [very complex formulas for large N, which depend on all $\boldsymbol{\eta} \in \mathbb{R}^N \times \mathbb{R}_+^N$ parameters]

Outline

Assumptions for parametric identification purposes



$$\frac{\mathrm{d}\boldsymbol{\beta}}{\mathrm{d}t} = \boldsymbol{S}_\beta(\boldsymbol{\beta}, \boldsymbol{\eta})\boldsymbol{u}_0$$

$$\boldsymbol{\eta} = \begin{bmatrix} \boldsymbol{\eta}_1 \\ \vdots \\ \boldsymbol{\eta}_N \end{bmatrix} \in \mathbb{R}^{2N}$$

A0: Unicycle-like (differentially-driven) or car-like tractor is used.

A1: A car-like tractor parameter $L_0 > 0$ is perfectly known (if applicable).

A2: Joint angles $\beta_1, \ldots, \beta_N$ [and steering angle β_0] are measured.

A3: Component v_0 of the tractor's velocity $\boldsymbol{u}_0 = [\omega_0 \ v_0]^\top$ is known, and:

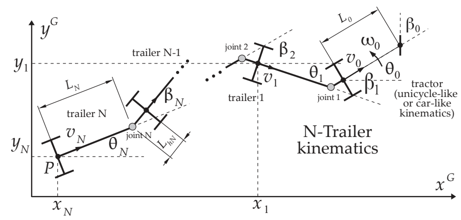
- ω_0 is directly available for the unicycle-like tractor
- ω_0 can be reconstructed for a car-like tractor upon A1 and A2:

$$\omega_0 = \tan(\beta_0) v_0 / L_0.$$

A4: A finite set $\mathrm{Z}^M \triangleq \{\boldsymbol{u}_0(nT_p), \boldsymbol{\beta}(nT_p)\}_{n=0}^{M-1}$ of data is available for computational purposes ($T_p = \text{const} > 0$ is a sampling interval).

Parameters in the vector $\boldsymbol{\eta}$ are uncertain or unknown.

Parametric estimation for kinematic models of N-trailers – problem statement



$$\frac{\mathrm{d}\boldsymbol{\beta}}{\mathrm{d}t} = \boldsymbol{S}_{\beta}(\boldsymbol{\beta}, \boldsymbol{\eta})\boldsymbol{u}_0$$

$$\boldsymbol{\eta} = \begin{bmatrix} \boldsymbol{\eta}_1 \\ \vdots \\ \boldsymbol{\eta}_N \end{bmatrix} \in \mathbb{R}^{2N}$$

Upon assumptions A0-A5:

- find a scalable* procedure for generic kinematics (1) which
- enables estimating parameters $\boldsymbol{\eta}_i$, $i = 1, \ldots, N$
- for arbitrary number N of trailers and arbitrary types of hitching (leading to the SNT, GNT, or nSNT kinematics)
- by using only the available finite set Z^M of data.

* Scalability is understood here with respect to N.

Outline

Iterative linear regression form of joint-angle kinematics

Thanks to the cascade nature of model (1), one can rewrite the ith row of (1) in the iterative form:

$$\begin{aligned}\beta_i^{(1)} &= \boldsymbol{c}^\top \boldsymbol{\Gamma}_i(\beta_j, \boldsymbol{\eta}_j) \prod_{j=i-1}^{1} \boldsymbol{J}_j(\beta_j, \boldsymbol{\eta}_j)\boldsymbol{u}_0 \\ &= \boldsymbol{c}^\top \boldsymbol{\Gamma}_i(\beta_j, \boldsymbol{\eta}_j)\boldsymbol{u}_{i-1} \\ &= [1\ 0]\left(\boldsymbol{I}_{2\times 2} - \boldsymbol{J}_i(\beta_j, \boldsymbol{\eta}_j)\right)\begin{bmatrix}\omega_{i-1}\\ v_{i-1}\end{bmatrix} \\ &= \left(1 + \frac{L_{hi}}{L_i}\cos\beta_i\right)\omega_{i-1} - \frac{1}{L_i}\sin\beta_i v_{i-1} \qquad \text{for} \quad i = 1, \ldots, N, \end{aligned} \tag{3}$$

where for fixed wheels holds

$$\boldsymbol{J}_i(\beta_i, \boldsymbol{\eta}_i) = \begin{bmatrix} -\frac{L_{hi}}{L_i}\cos\beta_i & \frac{1}{L_i}\sin\beta_i \\ L_{hi}\sin\beta_i & \cos\beta_i \end{bmatrix}. \tag{4}$$

Iterative linear regression form of joint-angle kinematics (cont.)

By defining an auxiliary output

$$y_i(t) \triangleq \beta_i^{(1)}(t) - \omega_{i-1}(t), \tag{5}$$

the model (3), recalled here as

$$\beta_i^{(1)} = \left(1 + \frac{L_{hi}}{L_i}\cos\beta_i\right)\omega_{i-1} - \frac{1}{L_i}\sin\beta_i v_{i-1}, \qquad i = 1, \ldots, N, \tag{6}$$

Iterative linear regression form of joint-angle kinematics (cont.)

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can be rewritten in the linear-regression form

$$y_i(t) = \frac{L_{hi}}{L_i}\cos\beta_i(t)\omega_{i-1}(t) - \frac{1}{L_i}\sin\beta_i(t)v_{i-1}(t) \quad \Rightarrow \quad \boxed{y_i(t) = \underbrace{[\varphi_{ai}(t) \;\; \varphi_{bi}(t)]}_{\boldsymbol{\phi}_i^\top(t)} \boldsymbol{p}_i} \tag{7}$$

with

$$\boldsymbol{\phi}_i(t) = \begin{bmatrix}\varphi_{ai}(t)\\ \varphi_{bi}(t)\end{bmatrix} = \begin{bmatrix}\cos\beta_i(t)\omega_{i-1}(t)\\ -\sin\beta_i(t)v_{i-1}(t)\end{bmatrix} \in \mathbb{R}^2 \quad \text{and} \quad \boldsymbol{p}_i = \begin{bmatrix}p_{ai}\\ p_{bi}\end{bmatrix} = \begin{bmatrix}\frac{L_{hi}}{L_i}\\ \frac{1}{L_i}\end{bmatrix} = \begin{bmatrix}\eta_{1i}/\eta_{2i}\\ 1/\eta_{2i}\end{bmatrix} \in \mathbb{R} \times \mathbb{R}_+. \tag{8}$$

Iterative linear regression form of joint-angle kinematics (cont.)

An iterative form of the linear regression

$$y_i(t) = \left[\cos\beta_i(t)\omega_{i-1}(t) \quad -\sin\beta_i(t)v_{i-1}(t)\right]\boldsymbol{p}_i, \qquad \boldsymbol{p}_i = \begin{bmatrix}p_{ai}\\ p_{bi}\end{bmatrix} = \begin{bmatrix}\eta_{1i}/\eta_{2i}\\ 1/\eta_{2i}\end{bmatrix} \tag{9}$$

requires

- velocities ω_0 and v_0 for $i = 1 \rightarrow$ available upon assumption A3,
- velocities ω_{i-1} and v_{i-1} for $i \geq 2 \rightarrow$ not available directly, but they can be replaced by predicted values:

$$\bar{\omega}_{i-1} \overset{(2)}{=} \boldsymbol{c}^\top \prod_{j=i-1}^{1} \boldsymbol{J}_j(\beta_j, \hat{\boldsymbol{\eta}}_j)\boldsymbol{u}_0, \qquad \boldsymbol{c}^\top \triangleq [1 \; 0] \tag{10}$$

$$\bar{v}_{i-1} \overset{(2)}{=} \boldsymbol{d}^\top \prod_{j=i-1}^{1} \boldsymbol{J}_j(\beta_j, \hat{\boldsymbol{\eta}}_j)\boldsymbol{u}_0, \qquad \boldsymbol{d}^\top \triangleq [0 \; 1], \tag{11}$$

where

$$\boldsymbol{J}_j(\beta_j, \hat{\boldsymbol{\eta}}_j) = \begin{bmatrix}-\frac{\hat{L}_{hj}}{\hat{L}_j}\cos\beta_j & \frac{1}{\hat{L}_j}\sin\beta_j\\ \hat{L}_{hj}\sin\beta_j & \cos\beta_j\end{bmatrix} \quad \text{for} \quad j = 1, \ldots, i-1, \qquad \hat{\boldsymbol{\eta}}_j = \begin{bmatrix}\hat{p}_{aj}/\hat{p}_{bj}\\ 1/\hat{p}_{bj}\end{bmatrix} = \hat{\boldsymbol{\eta}}_j(\hat{\boldsymbol{p}}_j) \tag{12}$$

Linear regression in a **data-explanatory** (stochastic) form

If the samples of joint-angles (assumption A2) are corrupted by stochastic noises $\xi_i(nT_p)$, that is,

$$\bar{\beta}_i(nT_p) \;=\; \underbrace{\beta_i(nT_p)}_{\text{true value}} + \underbrace{\xi_i(nT_p)}_{\text{noise}}, \qquad i = 1, \ldots, N, \tag{13}$$

and because the velocities are either noisy (for $i = 1$) or perturbed (for $i > 1$ due to prediction (10)-(11)), i.e.:

$$\bar{\omega}_{i-1}(nT_p) = \underbrace{\omega_{i-1}(nT_p)}_{\text{true value}} + \underbrace{\rho_{i-1}(nT_p)}_{\text{perturbation}}, \qquad \bar{v}_{i-1}(nT_p) = \underbrace{v_{i-1}(nT_p)}_{\text{true value}} + \underbrace{\nu_{i-1}(nT_p)}_{\text{perturbation}},$$

Parametric estimation procedure

Linear regression in a **data-explanatory** (stochastic) form

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the model (9) should be rewritten in a **data-explanatory** form

$$y_i(nT_p) = \underbrace{[\bar{\varphi}_{ai}(nT_p) \quad \bar{\varphi}_{bi}(nT_p)]}_{\boldsymbol{\varphi}_i^\top(nT_p)} \boldsymbol{p}_i + \zeta_i(nT_p), \qquad i = 1, \ldots, N, \tag{14}$$

where ζ_i represents a resultant (unmeasurable) disturbance of a linear-regression model, and now:

$$\boldsymbol{\varphi}_i^\top(nT_p) = [\bar{\varphi}_{a1}(nT_p) \quad \bar{\varphi}_{b1}(nT_p)] = [\cos\bar{\beta}_i(nT_p)\bar{\omega}_{i-1}(nT_p) \quad -\sin\bar{\beta}_i(nT_p)\bar{v}_{i-1}(nT_p)].$$

M. M. Michałek (IAR-PUT) Parametric estimation procedure for kinematics of N-trailers ZUT, Szczecin, 26.04.2024 16 / 29

Practical linear regression after (approximated) SVF filtration

The linear regression (14) cannot be used in practice because $y_i(t) \triangleq \beta_i^{(1)}(t) - \omega_{i-1}(t)$ is not available.

Parametric estimation procedure

Practical linear regression after (approximated) SVF filtration

The linear regression (14) cannot be used in practice because $y_i(t) \triangleq \beta_i^{(1)}(t) - \omega_{i-1}(t)$ is not available.

Therefore, one proposes to use the SVF (State Variable Filter(s) [GW:08]):

$$F^j(s) \triangleq \frac{s^j}{(1+sT_F)^k}, \qquad j \in \{0,1\}, \qquad k \geq 1, \qquad T_F := \eta T_p, \quad \eta > 1 \tag{15}$$

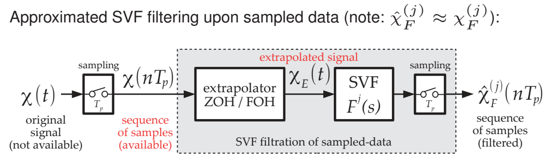
leading to the **practically useful** linear-regression form:

$$\boxed{\bar{y}_{iF}(nT_p) = \underbrace{[\bar{\varphi}_{aiF}(nT_p) \quad \bar{\varphi}_{biF}(nT_p)]}_{\boldsymbol{\varphi}_i^\top(nT_p)} \boldsymbol{p}_i + \bar{\zeta}_{iF}(nT_p),} \tag{16}$$

where

$$\bar{\varphi}_{aiF}(nT_p) = \mathcal{L}^{-1}\left\{F^0(s)[\bar{\varphi}_{ai}(t)]\right\}\big|_{t=nT_p}$$
$$\bar{\varphi}_{biF}(nT_p) = \mathcal{L}^{-1}\left\{F^0(s)[\bar{\varphi}_{bi}(t)]\right\}\big|_{t=nT_p}$$
$$\bar{y}_{iF}(nT_p) = \mathcal{L}^{-1}\left\{F^1(s)[\bar{\beta}_i(t)] - F^0(s)[\bar{\omega}_{i-1}(t)]\right\}\big|_{t=nT_p}$$

Approximated SVF filtering upon sampled data (note: $\hat{\chi}_F^{(j)} \approx \chi_F^{(j)}$):



[GW:08] H. Garnier, L. Wang (eds.): Identification of continuous-time models from sampled data, Springer-Verlag, London, 2008

Application of the LS estimator to the data-explanatory model

Now, one can apply a conventional Least Squares (LS) estimator

$$\hat{\boldsymbol{p}}_i = (\boldsymbol{\Phi}_i^\top \boldsymbol{\Phi}_i)^{-1} \boldsymbol{\Phi}_i^\top \bar{\boldsymbol{y}}_{iF}, \qquad \boldsymbol{\Phi}_i = \left[\boldsymbol{\varphi}_i(0)\ \boldsymbol{\varphi}_i(T_p)\ \ldots\ \boldsymbol{\varphi}_i((M-1)T_p)\right]^\top, \quad \bar{\boldsymbol{y}}_{iF} = \left[\bar{y}_{iF}(0)\ \ldots\ \bar{y}_{iF}((M-1)T_p)\right]^\top$$

using the sampled data according to the derived model structure

$$\bar{y}_{iF}(nT_p) = \underbrace{[\bar{\varphi}_{aiF}(nT_p) \quad \bar{\varphi}_{biF}(nT_p)]}_{\boldsymbol{\varphi}_i^\top(nT_p)} \boldsymbol{p}_i + \bar{\zeta}_{iF}(nT_p). \tag{17}$$

Parametric estimation procedure

Application of the LS estimator to the data-explanatory model

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$$\hat{\boldsymbol{p}}_i = (\boldsymbol{\Phi}_i^\top \boldsymbol{\Phi}_i)^{-1} \boldsymbol{\Phi}_i^\top \bar{\boldsymbol{y}}_{iF}, \qquad \boldsymbol{\Phi}_i = \left[\boldsymbol{\varphi}_i(0)\ \boldsymbol{\varphi}_i(T_p)\ \ldots\ \boldsymbol{\varphi}_i((M-1)T_p)\right]^\top, \quad \bar{\boldsymbol{y}}_{iF} = \left[\bar{y}_{iF}(0)\ \ldots\ \bar{y}_{iF}((M-1)T_p)\right]^\top$$

using the sampled data according to the derived model structure

$$\bar{y}_{iF}(nT_p) = \underbrace{[\bar{\varphi}_{aiF}(nT_p) \quad \bar{\varphi}_{biF}(nT_p)]}_{\boldsymbol{\varphi}_i^\top(nT_p)} \boldsymbol{p}_i + \bar{\zeta}_{iF}(nT_p). \tag{17}$$

In practice, one shall expect that:

- the disturbance $\bar{\zeta}_{iF}$ is autocorrelated (but with attenuated high-frequency components thanks to SVF)
- the regression variables $\bar{\varphi}_{aiF}$ and $\bar{\varphi}_{biF}$ are correlated with the disturbance $\bar{\zeta}_{iF}$
- the approximated SVF filtration is an additional source of a bias of the LS estimator [HW:20].

Consequence: the LS estimator applied to (17) will be biased,
but the bias can be acceptably small for a large number M of data samples in the set Z^M...

[HW:20] X.-L. Hu, J. S. Welsh: Continuous-time model identification from filtered sampled data: error analysis, IEEE TAC, 2020

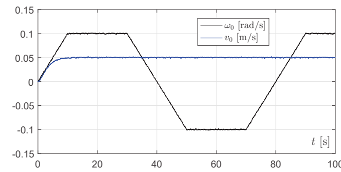
Outline

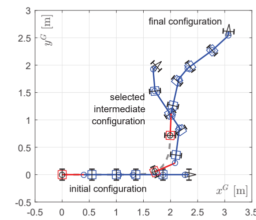
Why a nonholonomic G5T vehicle model?

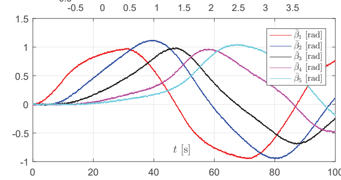
Application of the estimation procedure with the nonholonomic G5T vehicle model allows verification of the approach in the **nominal** conditions were **nonholonomic constraints are satisfied** by construction (no skid-slip effects can occur $\Rightarrow$ **a kinematic model structure is correct**).

Excitation of the G5T vehicle and assumed computational conditions [M:24]

- True parameters of the G5T vehicle (values in [m]):
  - $L_{h1} = 0.08,\ L_{h2} = 0.0,\ L_{h3} = 0.06,\ L_{h4} = -0.05,\ L_{h5} = 0.15$
    (hitching types: OFF+ / ON / OFF+ / OFF− / OFF+)
  - $L_1 = 0.4,\ L_2 = 0.5,\ L_3 = 0.3,\ L_4 = 0.5,\ L_5 = 0.4$
- Measurement and open-loop excitation conditions:
  - $T_p = 0.01\,\mathrm{s}$
  - $\bar{\beta}_i(nT_p) = \beta_i(nT_p) + \xi_i(nT_p),\ i = 1, \dots, 5$
  - $\xi_i(t) \triangleq \mathcal{L}^{-1}\{H(s)[e_{iE}(t)]\}, \quad H(s) = (1 + 0.08s)^{-1}$
  - $e_i \sim \mathcal{N}(0, 0.001)$
  - $\boldsymbol{u}_0(t)$: open-loop forward motion control perturbed by LPF Gaussian noises
- SVF (approximated) filtration parameters:
  - $F^j(s) = s^j/(1 + sT_F),\ j = 0, 1$
  - $T_F = 100T_p$
  - in Matlab: `lsim(F,x,t,'foh')`







[M:24] M. M. Michałek: Scalable parametric-identification procedure for kinematics of automated N-trailer vehicles, IEEE TVT, 2024

Selected estimation results for the G5T kinematics [M:24]

Means and standard deviations of estimators assessed upon 100 series of Z^{20001} data samples; the true values of parameters are highlighted in (blue):

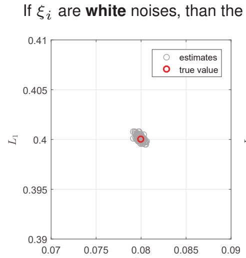
$\hat{L}_{h1} \pm \sigma_{h1}$	$\hat{L}_{h2} \pm \sigma_{h2}$	$\hat{L}_{h3} \pm \sigma_{h3}$	$\hat{L}_{h4} \pm \sigma_{h4}$	$\hat{L}_{h5} \pm \sigma_{h5}$
(+0.0800)	(0.0000)	(+0.0600)	(−0.0500)	(+0.1500)
+0.0799	−0.0000	+0.0602	−0.0498	+0.1501
±0.0003	±0.0007	±0.0009	±0.0009	±0.0018
$\hat{L}_1 \pm \sigma_1$	$\hat{L}_2 \pm \sigma_2$	$\hat{L}_3 \pm \sigma_3$	$\hat{L}_4 \pm \sigma_4$	$\hat{L}_5 \pm \sigma_5$
(+0.4000)	(+0.5000)	(+0.3000)	(+0.5000)	(+0.4000)
+0.4001	+0.5001	+0.3000	+0.4998	+0.3998
±0.0003	±0.0005	±0.0007	±0.0013	±0.0011

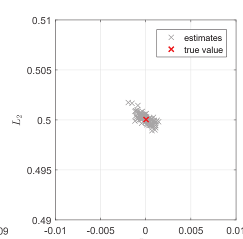
If ξ_i are **white** noises, than the resultant standard deviations become much smaller.

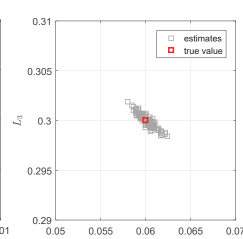
Recursive LS estimator was used with initial conditions:

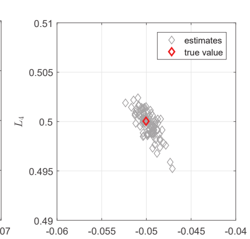
- $\hat{\boldsymbol{p}}_i(0) := \boldsymbol{0}$ for $i = 1, \dots, 5$
- $\boldsymbol{P}_i(0) := \mu_i \boldsymbol{I}_{2\times2}$, where: $\mu_1 = 10^4,\ \mu_2 = 5\mu_1,\ \mu_3 = 10\mu_1,\ \mu_4 = 20\mu_1,\ \mu_5 = 40\mu_1$
- estimates of vehicle parameters come from the equation:

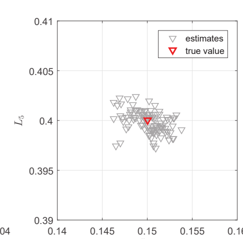
$$\hat{\boldsymbol{\eta}}_i = \begin{bmatrix} \hat{L}_{hi} \\ \hat{L}_i \end{bmatrix} = \begin{bmatrix} \hat{p}_{ai}/\hat{p}_{bi} \\ 1/\hat{p}_{bi} \end{bmatrix}$$





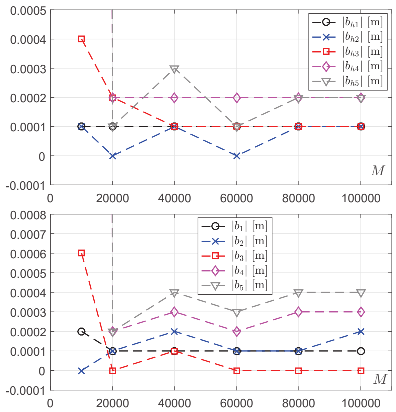






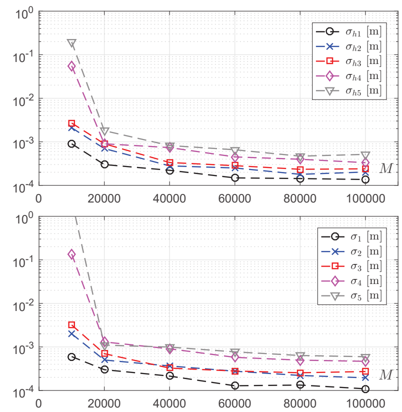
Verification of large-sample properties of the proposed estimator [M:24] (coloured noises ξ_i)

Ensemble means of absolute biases $b_{hi} \triangleq L_{hi} - \hat{L}_{hi}$ and $b_i \triangleq L_i - \hat{L}_i$, $i = 1, \ldots, 5$, as a function of a number M:



(each mean value computed upon the results of 100 estimation procedures)

Empirical standard deviations σ_{hi} and σ_i, $i = 1, \ldots, 5$, obtained upon $\hat{L}_{hi}$ and $\hat{L}_i$ as a function of a number M:



(each deviation computed upon the results of 100 estimation procedures)

Note: The plots are presented with the resolution of 10^{-4} m.

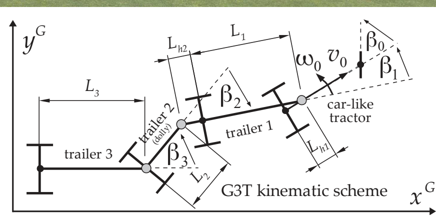
Outline

Why a high-fidelity TruckSim© vehicle model?

Application of the estimation procedure with the TruckSim vehicle model allows verification of the approach in **non-nominal** conditions when **nonholonomic constraints can be violated** (when the skid-slip effects occur $\Rightarrow$ **a kinematic model structure is perturbed**).

The selected TruckSim vehicle and the excitation experiment

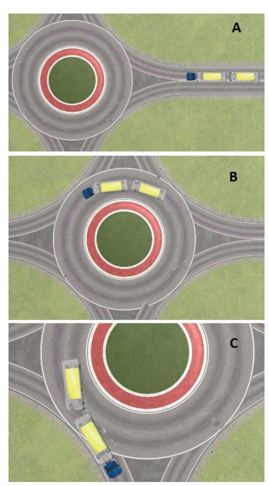
High-fidelity (kinetic) vehicle model from TruckSim©:



True kinematic parameters of the G3T vehicle (values in [m]):

- $L_{h1} = -0.600, \quad L_{h2} = 1.497, \quad L_{h3} = 0.000$
- $L_1 = 6.303, \quad L_2 = 1.980, \quad L_3 = 6.303$

Roundabout motion scenario for estimation-data collection:

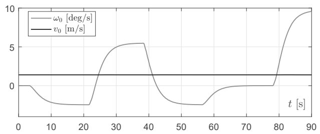


Estimation conditions:

- collected number of data samples $M = 140001$
- data sampling interval $T_p = 0.001$ s
- applied longitudinal speeds of a tractor $v_0 \in \{5, 10, 15\}$ km/h
- time constant applied for the SVF $T_F = 100T_p$
- the same RLS estimator, analogously initialized, was used for estimation purposes

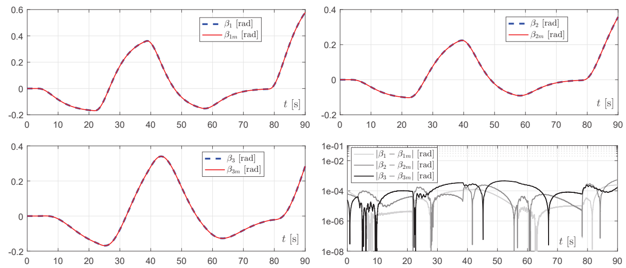
Estimation results and model validation for the TruckSim vehicle

Model validation for the new open-loop control excitation:



v_0 $\left[\frac{\text{km}}{\text{h}}\right]$	i	$\hat{L}_{hi}$ [m] (true value)	δ_i [%]	$\hat{L}_i$ [m] (true value)	Δ_i [%]	J^i_{FIT} [%]
5	1	-0.5921 (-0.6000)	1.3	6.3027 (6.3030)	0.004	99.96
	2	1.5201 (1.4970)	1.5	1.9553 (1.9800)	1.2	99.89
	3	0.0160 (0.0000)	singular	6.2915 (6.3030)	0.2	99.86
10	1	-0.5617 (-0.6000)	6.4	6.2892 (6.3030)	0.2	99.65
	2	1.6242 (1.4970)	8.5	1.8556 (1.9800)	6.3	98.97
	3	0.0670 (0.0000)	singular	6.2591 (6.3030)	0.7	99.09
15	1	-0.5121 (-0.6000)	14.6	6.2725 (6.3030)	0.5	98.97
	2	1.8098 (1.4970)	20.9	1.6887 (1.9800)	14.7	97.15
	3	0.1721 (0.0000)	singular	6.1900 (6.3030)	1.8	97.46

Validation results for the best model from the table:



Measures for quantitative model-quality assessment:

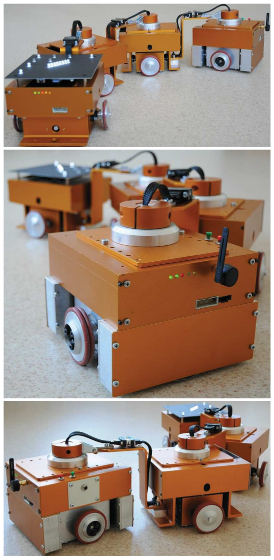
$$J^i_{\text{FIT}} \triangleq \left(1 - \frac{\|\boldsymbol{\beta}_i - \boldsymbol{\beta}_{im}\|}{\|\boldsymbol{\beta}_i - \text{m}[\boldsymbol{\beta}_i]\cdot\mathbf{1}\|}\right)100\%$$

$$\delta_i \triangleq \frac{\left||L_{hi}| - |\hat{L}_{hi}|\right|}{|L_{hi}|}100\%, \qquad \Delta_i \triangleq \frac{\left|L_i - \hat{L}_i\right|}{L_i}100\%$$

Number of validation data samples used: $M = 90000$

Outline

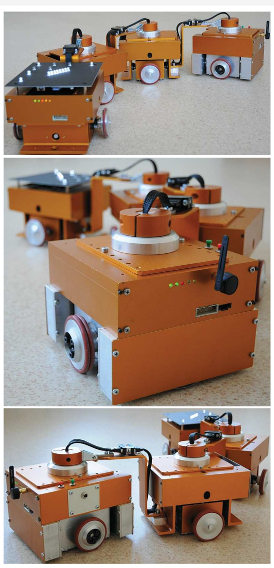
Final remarks and properties of the proposed estimation procedure



- $2N$-dimensional estimation problem reduced to N of 2-dimensional problems
- Sequential procedure – scalable with respect to the number N of trailers
- Applicable to N-Trailer kinematics of a any type (SNT, GNT, nSNT)
- No model discretization (continuous-time model identification from sampled data)
- A bias of the LS estimator and its decreasing precision for increasing $i \in [1; N]$
- Despite limitations, the large-sample estimator's properties seem acceptable

Conclusions

Final remarks and properties of the proposed estimation procedure



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Thank you for attention

M. M. Michałek (IAR-PUT) Parametric estimation procedure for kinematics of N-trailers ZUT, Szczecin, 26.04.2024 29 / 29

Appendix

Application of the procedure in the wheels-slippage conditions

- Consequences of nonholonomic constraints violation:
  - Kinematic model structure will not match the real system kinematics under conditions of the wheels-slippage:
    - lateral and longitudinal slippage of wheels possible in practice
    - measuring of lateral velocity components is difficult in practice
    - interpretation of kinematic parameters must be changed (*reduced* parameters)
  - In the case of fixed-multi-axle trailers when cornering:
    - reduced kinematic parameters can be time-varying due to a non-constant slippage of the wheels (unknown variability)
    - application of the RLS with a forgetting factor or Kalman estimator?

source: https://www.wikiwand.com

Open questions

- Identifiability $\longrightarrow$ persistent excitation (PE) conditions:
  - How should the control inputs $\boldsymbol{u}_0$ be selected to guarantee the PE conditions?
  - How do the number and lengths of the trailers affect PE conditions? (note: trailer-mobility index)
  - Does an application of feedback control in automated N-trailers affect identifiability?
- Bias elimination:
  - Is it worth applying the Instrumental Variable (IV) method instead of LS?
  - If yes, how to construct the instruments to minimise a variance of the IV estimator?